

Title: Well Babe

Brief Overview:

This unit is a means of studying and comparing the points of concurrency using Geometer's Sketchpad or CABRI. Students will use technology to find four points of concurrency in a triangle. The students will then decide which point is in the best position for a well that will serve three farms.

Links to NCTM 2000 Standards:

- **Mathematics as Problem Solving, Reasoning and Proof, Communication, Connections, and Representation**

These five process standards are threads that integrate throughout the unit, although they may not be specifically addressed in the unit. They emphasize the need to help students develop the processes that are the major means for doing mathematics, thinking about mathematics, understanding mathematics, and communicating mathematics.

Students will use Geometer's Sketchpad software to compare, interpret, and analyze the four points of concurrency. They will use correct mathematical language to describe the results of the comparison of the four points of concurrency and write about their findings. Students also will use their knowledge to interpret real-life situations.

- **Geometry and Spatial Sense**

Students will analyze characteristics and properties of two-dimensional geometric forms.

- **Measurement**

Students will measure the distance from the point of concurrency to the vertices of the triangle using technology or geometric tools.

Links to Maryland High School Mathematics Core Learning Units:

Geometry, Measurement, and Reasoning

- **2.1.1**

Students will describe the characteristics of geometric figures and will construct or draw geometric figures using technology and tools.

- **2.1.4**

Students will validate properties of geometric figures using appropriate tools and technology.

- **2.3.1**

Students will use algebraic and geometric properties to measure indirectly.

Grade/Level:

Grades 9-12; Geometry

Duration/Length:

2 - 45 minute periods

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Construction of an altitude, perpendicular bisector, angle bisector and median
- Knowledge of the orthocenter, circumcenter, incenter, and centroid of a given triangle
- Basic skills and usage of the Geometer's Sketchpad or CABRI software

Student Outcomes:

Students will be able to:

- find four points of concurrency for any given triangle.
- interpret and compare results to determine the best location for the well.

Materials/Resources/Printed Materials:

- Geometer's Sketchpad or CABRI software
- Student Activity Sheet
- Student Assessment

Development/Procedures:

- Teacher will review definitions of the points of concurrency.
- Teacher will review any necessary computer procedures.
- Students will use the activity sheet to explore the points of concurrency.
- Students will print their solution for the best position of the well.
- An assessment is provided to conclude the activity.

Assessment:

An assessment activity and scoring rubric are provided.

Extension/Follow Up:

- Similar activities using points of concurrency for other polygons can be developed through the use of technology.
- Students should be encouraged to experiment with other polygons and analyze their findings.
- Students could find the best location for a stadium within the boundaries of three major highways. (The incenter is equidistant from sides of a triangle.)

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References:

Discovering Geometry, by Michael Serra, Key Curriculum Press, 1993
Geometer's Sketchpad, Version 3.0

Teacher Notes: Well Babe

Purpose:

Students will be able to explore real-life situations using points of concurrency.

Previous Experience:

Teacher needs to be familiar with Geometer's Sketchpad or CABRI software.

Materials Needed:

- Geometer's Sketchpad or CABRI software
- Student Activity Sheet
- Student Assessment

Notes to Teacher:

**This lesson may be done without the use of Geometer's Sketchpad or CABRI software. More time will be needed for the students to construct points of concurrency with a compass and straightedge.

General Steps for Sketchpad Activity:

(Basic familiarity with Geometer's Sketchpad is needed for this activity)

1. Follow student activity sheet directions for set-up of problem on computer.
2. a. Find the intersection of the medians.

Step 1- Select all three sides of the triangle by clicking and holding shift key.

Step 2- Click *Construct*, then *Point at midpoint*.

Step 3- Construct 3 line segments from each vertex to the opposite sides midpoint.

Step 4- Select two of the segments, Click *Construct*, then *Point at intersection*.

Step 5- Measure the 3 segments formed between the centroid and the vertices.

- b. Find the intersection of the altitudes.

(before starting part b, students may want to clear the triangle of segments made from part a.)

Step 1- Select a vertex and opposite side.

Step 2- Click *Construct*, then *Perpendicular Line*.

Step 3- Repeat steps 1 and 2 for the other vertices and sides.

Step 4- Construct the point at intersection of these lines. (refer to part a)

Step 5- Measure the 3 segments formed between the orthocenter and the vertices.

c. Find the intersection of the angle bisectors.

Step 1- Select all 3 vertices and create angle bisectors for each angle.

Step 2- Construct the point at intersection of these lines. (refer to part a)

Step 3- Measure the 3 segments formed between the incenter and the vertices.

d. Find the intersection of the perpendicular bisectors.

Step 1- Find the midpoints of each side.

Step 2- Construct a perpendicular line through the midpoint of each side.

Step 3- Construct the point at intersection of these lines. (refer to part a)

Step 4- Construct segments from each vertex to the circumcenter.

Step 4- Measure the 3 segments formed between the circumcenter and the vertices.

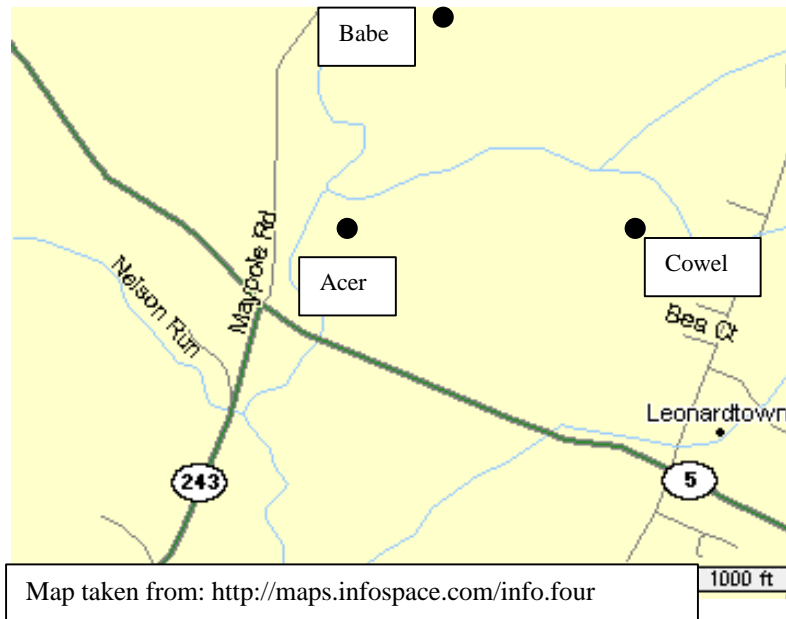
3. Students will discover that the circumcenter is the solution for the best position of the well because it is equidistant from the vertices. Teacher may have to guide some students toward this answer.
4. Students will discover that the circumcenter is the center of a circumscribed circle. This question can be used for students who finish questions 1-3 early. Students can find the answer by experimenting with drawing circles with the different points of concurrency as the center.

Name _____
Date _____ Period _____

Student Activity Sheet – “Well Babe”

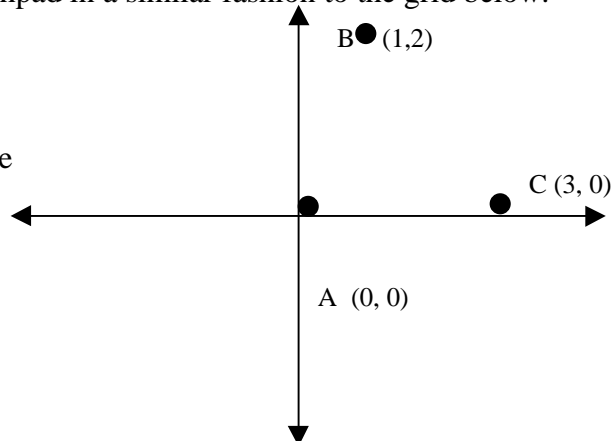
Scenario:

Well Babe, a farmer in St. Mary’s County, wanted to share a well for water use with his neighbors, Acer and Cowel. The other farmers agreed to share and they decided to locate the well equidistant from each farm. You have been called as the contractor to locate the best position for the well. You need to justify to your clients the position of the well.



1. Transpose the problem to the sketchpad in a similar fashion to the grid below.

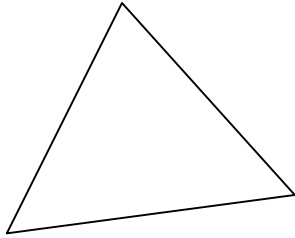
- Click on Graph, then Show Axes.
- Click on Graph, then Show Grid.
- Graph the three points given in the figure at the right.



2. Draw the indicated segments on your Sketchpad triangle (created from problem 1).
Calculate the lengths of the segments that join the vertices and the point of concurrency. Write a conjecture about your observations.

a. Find the intersection of the medians.

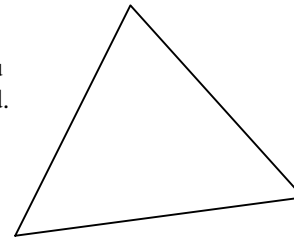
Sketch of
drawing that you
see in Sketchpad.



Conjecture:

b. Find the intersection of the altitudes.

Sketch of
drawing that you
see in Sketchpad.

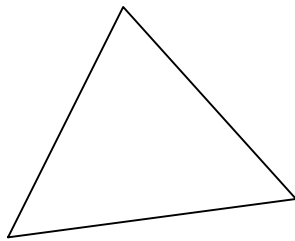


Measurements

Conjecture:

c. Find the intersection of the angle bisectors.

Sketch of
drawing that you
see in Sketchpad.

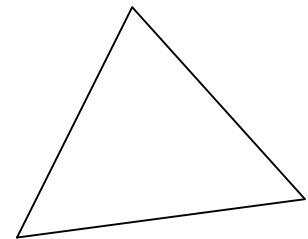


Conjecture:

Measurements:

d. Find the intersection of the perpendicular bisectors.

Sketch of
drawing that you
see in Sketchpad.



Conjecture:

Measurements:

3. Determine the best position for the well. Justify your answer.

4. Challenge: Which point of concurrency would you use to circumscribe a circle?
(circle on the outside of the triangle)

Assessment

Teacher's Guide

Introduction

This assessment is to follow “Well Babe”. There are two parts to the assessment. Part I is a selected response and Part II is a brief constructed response. Geometer's Sketchpad or CABRI is not to be used for the assessment.

Objectives Covered

This assessment assumes your students have received instruction on the following objectives:

- Find the four points of concurrency for any given triangle
- Interpret and compare points of concurrency that are used in real life situations.

Tools/Materials Needed for Assessment

- Pencil
- Protractor or compass and straight edge
- Copies of assessment

Administering the Assessment

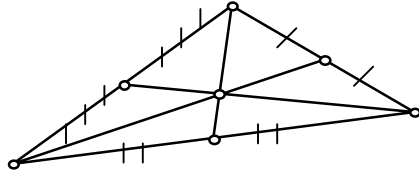
This assessment should take approximately 20 minutes. Distribute the papers and any necessary material.

Student Assessment

Part I Selected Response

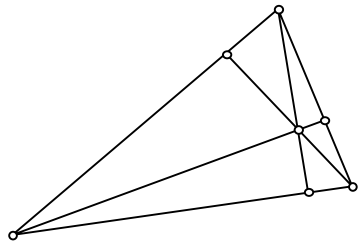
Read each problem carefully and circle the appropriate answer.

1. The point of concurrency in the figure is:



- a. centroid b. circumcenter c. incenter d. orthocenter

2. The point of concurrency in the figure is:



- a. centroid b. circumcenter c. incenter d. orthocenter

3. A line segment with one endpoint on a vertex and another endpoint on a midpoint of the opposite side is:

- a. angle bisector b. altitude c. median d. perpendicular bisector

4. A line segment from a vertex and perpendicular to the opposite side is:

- a. angle bisector b. altitude c. median d. perpendicular bisector

5. The point of concurrency of the bisectors of the interior angles of a triangle is:

- a. centroid b. circumcenter c. incenter d. orthocenter

6. The point of concurrency of the perpendicular bisectors of each side of a triangle is the following:
- a. centroid b. circumcenter c. incenter d. orthocenter

Part II Brief Constructed Response

Read the following scenario. Justify your solution by drawing the appropriate segments. Use a protractor or compass and straightedge. Label and mark your diagram.

Scenario:

Paul Hines owns a 60-acre buffalo farm in Churchville, Maryland (National Geographic, Nov. 1994). He wants to move his feeding trough to make it easily accessible to his entire herd. His property is triangular. Mr. Hines doesn't want any angry buffalo so he wants the trough equidistant from the vertices of the triangle. You are asked to find the position for the feeding trough and to justify why you placed the trough in that position.

Picture taken from: <http://www.narvonvalleybison.com>

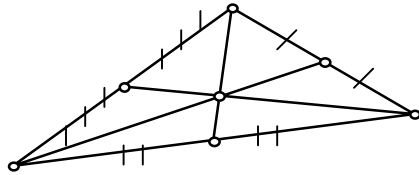


Student Assessment Key

Part I Selected Response

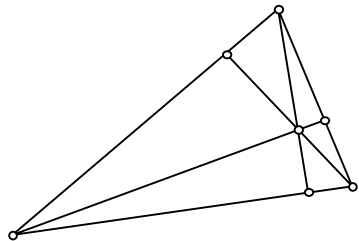
Read each problem carefully and circle the appropriate answer.

1. The point of concurrency in the figure is:



- a. centroid b. circumcenter c. incenter d. orthocenter

2. The point of concurrency in the figure is:



- a. centroid b. circumcenter c. incenter d. orthocenter

3. A line segment with one endpoint on a vertex and another endpoint on a midpoint of the opposite side is:

- a. angle bisector b. altitude c. median d. perpendicular bisector

4. A line segment from a vertex and perpendicular to the opposite side is:

- a. angle bisector b. altitude c. median d. perpendicular bisector

5. The point of concurrency of the bisectors of the interior angles of a triangle is:

- a. centroid b. circumcenter c. incenter d. orthocenter

6. The point of concurrency of the perpendicular bisectors of each side of a triangle is the following:

- a. centroid **b. circumcenter** c. incenter d. orthocenter

Part II Brief Constructed Response

Read the following scenario. Justify your solution by drawing the appropriate segments. Use a protractor or compass and straightedge. Label and mark your diagram.

Scenario:

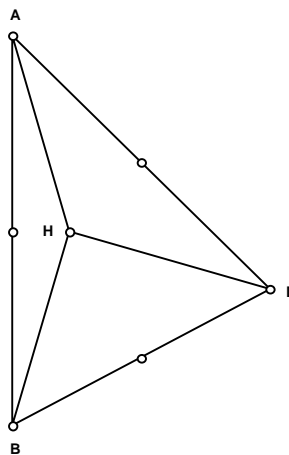
Paul Hines owns a 60-acre buffalo farm in Churchville, Maryland (National Geographic, Nov. 1994). He wants to move his feeding trough to make it easily accessible to all his herd. His property is triangular. Mr. Hines doesn't want any angry buffalo so he wants the trough equidistant from the vertices of the triangle. You are asked to find the position for the feeding trough and to justify why you placed the trough in that position.

Picture taken from: <http://www.narvonvalleybison.com>



The circumcenter is the point of concurrency that should be used to determine the location of the feeding trough. This point is equidistant from the three vertices.

$$HA = HB = HD$$



Scoring Rubric

- 4 Construction of three perpendicular bisectors with right angle labeled and midpoint labeled. Three measurements from point of concurrency to vertices are accurate. Clear statement that circumcenter is equidistant from the vertices.
- 3 Construction of the three perpendicular bisectors with right angle labeled or midpoint labeled. Three measurements from points of concurrency to vertices are close to correct response. Somewhat clear statement that circumcenter is equidistant from the vertices.
- 2 Construction of three perpendicular bisectors but neither right angle labeled nor midpoint labeled. No measurements or inaccurate measurements shown. Vague statement showing that circumcenter is equidistant from vertices.
- 1 Incomplete or inaccurate construction. Incomplete or inaccurate measurements. No statement describing circumcenter.
- 0 No response.